Georgia Assessments for the Certification of Educators®

TEST DESIGN AND FRAMEWORK

TEST DESIGN

Mathematics

The **Mathematics** assessment consists of **two tests**. Each test contains a section with selected-response questions and a section with constructed-response assignments. Each section counts for a percentage of your total test score. The areas of content assessed by each test, the approximate number of selected-response questions and constructed-response assignments in each content area, and the percentage of your total test score derived from each test section are shown in the tables below. Further information regarding the content included in each subarea can be found in the test framework.

Test I (Test Code 022)

Subareas:	Objectives	Approximate Number of Selected-Response Questions	Constructed-Response Assignments
➢ Number Concepts and Operations	0001–0002	16	
➢ Algebra	0003–0005	22	1
➢ Precalculus and Calculus	0006–0008	22	1
	TOTAL	**60**	**2**
	Percentage of Test Score	**80%**	**20%**

Test II (Test Code 023)

Subareas:	Objectives	Approximate Number of Selected-Response Questions	Constructed-Response Assignments
➢ Geometry and Measurement	0009–0011	22	1
➢ Data Analysis and Probability	0012–0014	22	1
➢ Mathematical Processes and Perspectives	0015–0016	16	
	TOTAL	**60**	**2**
	Percentage of Test Score	**80%**	**20%**

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TEST DESIGN AND FRAMEWORK

TEST FRAMEWORK

Mathematics

NUMBER CONCEPTS AND OPERATIONS

0001 Understand number operations and basic principles of number theory.

For example:

- analyzing and applying properties of positive and negative numbers (e.g., absolute value)
- analyzing number operations related to rational numbers
- demonstrating knowledge of basic concepts from number theory (e.g., greatest common divisor, least common multiple)
- applying the fundamental theorem of arithmetic to determine prime factorization of numbers
- solving problems using a variety of representations of numbers (e.g., fractions, decimals, percents, exponents, scientific notation)

0002 Understand the real and complex number systems.

For example:

- analyzing and applying properties of the real number system (e.g., distributive, associative)
- analyzing number operations (e.g., roots, laws of exponents) related to real numbers
- identifying appropriate representations of complex numbers (e.g., rectangular, polar, vector, matrix) for particular situations
- analyzing, simplifying, and applying operations (e.g., addition, multiplication, roots) to expressions involving complex numbers, including their trigonometric forms

ALGEBRA

0003 Understand algebraic operations and properties of functions and relations.

For example:

- simplifying, evaluating, and performing operations (e.g., factoring, expanding binomials using the binomial theorem) on polynomials and other algebraic expressions
- solving algebraic equations
- analyzing direct and inverse proportional relationships (e.g., constant of proportionality)
- identifying and analyzing characteristics of functions (e.g., domain, range, intercepts, extrema, symmetry) using graphs, tables, and algebraic techniques and relating those characteristics to real phenomena and everyday situations
- analyzing transformations (e.g., shifts, stretches, reflections) of functions and relations
- identifying and determining compositions and inverses of various functions (e.g., linear, exponential, rational) algebraically and graphically
- recognizing sequences as functions with domains that are whole numbers

0004 Understand properties of linear equations, inequalities, and systems.

For example:

- analyzing various representations (e.g., verbal, tabular, graphic, algebraic) of linear functions
- analyzing the relationship between a linear equation and its graph
- interpreting the meaning of slope and *y*-intercept in a given situation
- recognizing, interpreting, and solving problems involving linear equations and systems of linear equations algebraically, graphically, and using matrices
- solving problems involving linear inequalities and systems of linear inequalities using a variety of techniques (e.g., algebraic, graphic, linear programming)
- performing algebraic operations on vectors and matrices
- using matrices to model and solve problems
- demonstrating knowledge of vertex-edge graphs

0005 Understand properties of quadratic functions.

For example:

- analyzing the nature of the roots of quadratic equations
- solving problems involving quadratic equations and inequalities using a variety of techniques (e.g., algebraic, graphic) and appropriate technology
- analyzing various representations (e.g., tabular, graphic, algebraic) of quadratic functions
- analyzing the graphs and characteristics (e.g., vertex, axis of symmetry, zeroes) of quadratic functions in the forms $y = ax^2 + bx + c$ and $y = a(x – h)^2 + k$
- demonstrating knowledge of sequences of partial sums of arithmetic series as examples of quadratic functions

PRECALCULUS AND CALCULUS

0006 Understand properties of nonlinear functions.

For example:

- analyzing various representations of nonlinear (e.g., polynomial, rational, radical, exponential, logarithmic, absolute value, piecewise) functions
- demonstrating knowledge of properties of logarithms and the laws of exponents
- analyzing nonlinear functions in terms of discontinuities, asymptotes, intervals of increase or decrease, and end behavior
- solving equations and inequalities involving nonlinear functions using a variety of techniques (e.g., algebraic, graphic) and appropriate technology
- using nonlinear functions to model real phenomena (e.g., half-life) and everyday situations (e.g., area and volume models)
- recognizing and interpreting graphs of equations in rectangular, parametric, and polar forms

0007 Understand properties of trigonometric functions and identities.

For example:

- applying trigonometric ratios to right triangles
- applying the laws of sines and cosines to solve problems
- analyzing the relationships among the unit circle, circular functions, and trigonometric functions for angles measured in both degrees and radians
- identifying and analyzing the graphs of trigonometric functions and their transformations in terms of their properties (e.g., amplitude, period, phase shift)
- applying trigonometric functions in authentic contexts involving periodic phenomena
- recognizing the relationships between trigonometric functions and their inverses numerically, analytically, and graphically
- identifying and applying trigonometric identities
- solving trigonometric equations algebraically and graphically

0008 Understand principles and applications of calculus.

For example:

- analyzing sequences and series and their properties (e.g., convergence, limits)
- applying the concept of limit to difference quotients and Riemann sums
- applying properties of derivatives to analyze graphs of functions and solve problems (e.g., instantaneous rate of change, optimization)
- using integral calculus to represent and solve a variety of problems (e.g., area, volume, displacement, velocity)

GEOMETRY AND MEASUREMENT

0009 Understand the principles of measurement.

For example:

- identifying appropriate units for finding and expressing measurements (e.g., length, perimeter, area, density, speed)
- demonstrating knowledge of selecting and using appropriate measurement tools (e.g., ruler, protractor)
- converting from one unit to another within the customary and metric systems of measurement
- solving problems involving perimeter, area, surface area, or volume of geometric figures and shapes (e.g., polygons, circles, spheres, prisms, cones)
- analyzing various views (e.g., cross sections, nets) of three-dimensional shapes
- applying the concept of similarity, scale factors, and proportional reasoning to solve measurement problems

0010 Understand principles of Euclidean geometry.

For example:

- applying the language of mathematical argument (e.g., definition, axiom, theorem, converse of a statement, inverse of a statement)
- analyzing and applying properties related to points, lines, planes, and angles
- applying properties of similarity and congruence to solve problems and justify conclusions
- analyzing and applying properties of triangles (e.g., Pythagorean theorem, triangle inequality), quadrilaterals, and other polygons to solve problems and justify conclusions
- analyzing and applying properties of circles, lines that intersect circles (e.g., secants, tangents), and related angles to solve problems
- applying the properties of two- and three-dimensional figures to solve problems

0011 Understand coordinate and transformational geometry.

For example:

- identifying transformations (e.g., reflections, translations, rotations, dilations) of figures represented in the coordinate plane
- applying symmetry to explore plane figures and their properties
- using concepts and properties of slope, midpoint, parallelism, perpendicularity, and distance to explore properties of figures in the coordinate plane
- identifying, analyzing, and graphing equations of conic sections (e.g., circles, hyperbolas, ellipses, parabolas)
- plotting points and using the distance formula in 3-space
- identifying equations of planes and spheres in 3-space
- solving problems using vectors expressed using rectangular coordinates and vectors expressed using magnitude and direction

DATA ANALYSIS AND PROBABILITY

0012 Understand methods of collecting, organizing, and describing data.

For example:

- applying knowledge of collecting data using surveys, simulations, observations, and other tools
- demonstrating knowledge of how to design and conduct statistical experiments to investigate real-world problems
- demonstrating knowledge of techniques for organizing, displaying, and describing data in a variety of formats (e.g., tables, frequency distributions, histograms, circle graphs, bar graphs, line plots, scatter plots, stem-and-leaf plots, box-and-whisker plots)
- determining and interpreting statistical measures (e.g., mean, median, mode, range, variance, standard deviation, interquartile range, percentile)

0013 Understand the theory and applications of probability.

For example:

- applying addition and multiplication counting principles to determine the number of outcomes related to an event
- determining and using permutations and combinations to solve a variety of problems
- determining probabilities of simple and compound events (e.g., dependent, independent, mutually exclusive, conditional)
- analyzing different graphical representations (e.g., Venn diagrams, tree diagrams) to calculate and interpret probabilities
- calculating and using expected value to predict the outcome of an event
- demonstrating knowledge of methods for predicting the probability of an event using trials or simulations

0014 Understand the process of analyzing and interpreting data to make statistical inferences.

For example:

- demonstrating knowledge of the nature of sampling and the importance of random sampling
- applying the normal probability distribution (e.g., interpreting *z*-scores) to solve a variety of problems
- determining and analyzing linear and quadratic fits to data using a variety of methods (e.g., median-median line) and using linear and quadratic regression models with appropriate technology
- interpreting the relationship between two variables, including the difference between correlation and causation
- using summary statistics to make inferences about population parameters
- demonstrating knowledge of the law of large numbers and the central limit theorem
- determining and interpreting confidence intervals and margins of error

MATHEMATICAL PROCESSES AND PERSPECTIVES

0015 Understand how to use a variety of representations to communicate mathematical ideas and concepts and connections between them.

For example:

- communicating mathematical ideas used to demonstrate knowledge of a variety of representations (e.g., numerical, tabular, graphical, pictorial, symbolic)
- translating among algebraic, graphical, symbolic, diagrammatic, and other means of presenting mathematical ideas (e.g., set notation, interval notation, vectors)
- translating among mathematical language, notation, and symbols and everyday language
- recognizing precise and accurate use of mathematical terminology
- analyzing and evaluating a mathematical model (e.g., equation, graph, pictorial representation) in terms of its appropriateness and usefulness in a given situation
- recognizing connections among different concepts and areas of mathematics (e.g., algebra and geometry) and using them to solve problems
- recognizing and applying mathematics in contexts outside of mathematics

0016 Understand mathematical reasoning, the construction of mathematical arguments, and problem-solving strategies in mathematics and other contexts.

For example:

- demonstrating knowledge of the nature of mathematical argument, including direct and indirect proof, and the use of counterexamples
- applying correct mathematical reasoning to draw valid conclusions and evaluate arguments and proofs
- identifying inductive and deductive reasoning and using them to generate and investigate the validity of conjectures
- selecting an appropriate problem-solving strategy (e.g., estimation, working backward, drawing a diagram) for investigating or solving a particular problem
- demonstrating knowledge of problem solving using appropriate technology
- demonstrating knowledge of how mathematical concepts and strategies are applied across the curriculum and in everyday contexts to model and solve problems