



SECTION 2

TEST I SAMPLE QUESTIONS

This section of the Georgia Assessments for the Certification of Educators® (GACE™) Preparation Guide provides sample selected-response questions with an annotated answer key for you to review as part of your preparation for the test. The sample selected-response questions are designed to illustrate the nature of the test questions. Work through the questions carefully before referring to the annotated answer key, which follows the sample selected-response questions. The answer key provides the correct response to each question, describes why each correct response is the best answer, and lists the objective within the test framework to which each question is linked.

Please note that definitions and formulas are provided for this test. Please refer to these materials as needed in responding to the sample test questions and assignments. These materials are located in the Assessment Reference Materials section at the end of this preparation guide.

A graphing calculator may be used for this test as needed in responding to the sample test questions and assignments. Please refer to the current GACE registration bulletin for information about the use of calculators at the test administration.

QUESTIONS

- Given that r and p are positive integers, which of the following expressions must represent a rational number?
 - \sqrt{rp}
 - $\frac{r!}{p!}$
 - re^p
 - $rp\left(\frac{p}{r}\right)$

- Converting a complex number from the standard $a + bi$ form to the trigonometric $r(\cos \theta + i \sin \theta)$ form would be most helpful when:
 - graphing the complex number.
 - finding the conjugate of the complex number.
 - finding the absolute value of the complex number.
 - raising a complex number to a power.

- The period of a pendulum is directly proportional to the square root of the length of the arm of the pendulum. A pendulum with an arm length of 4 feet has a period of 2.2 seconds. What is the constant of proportionality?
 - 4.4
 - 1.1
 - 0.55
 - 0.14

4. Use the driving time matrix below to answer the question that follows.

Driving Time in Hours

	Albany, NY	Atlanta, GA	Austin, TX	Boston, MA	Chicago, IL	Denver, CO	Detroit, MI
Albany, NY	0	16	29	3	13	28	9
Atlanta, GA	16	0	14	18	11	21	11
Austin, TX	29	14	0	31	18	15	21
Boston, MA	3	18	31	0	16	30	12
Chicago, IL	13	11	18	16	0	15	4
Denver, CO	28	21	15	30	15	0	19
Detroit, MI	9	11	21	12	4	19	0

A delivery truck leaves Atlanta, makes a first stop in Boston, a second stop in Chicago, a third stop in Denver, and then returns to Atlanta. Based on the driving time matrix above, what is the total number of driving hours for the route?

- A. 71 hours
- B. 70 hours
- C. 67 hours
- D. 50 hours

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5. Use the information below to answer the question that follows.

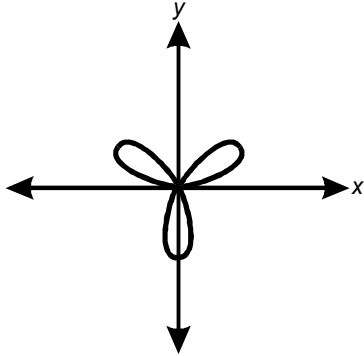
$$\begin{aligned}f(x) &= ax^2 + bx + c \\b^2 - 4ac &= 3 \\a &\neq 0\end{aligned}$$

The information above implies that the function $f(x)$ has:

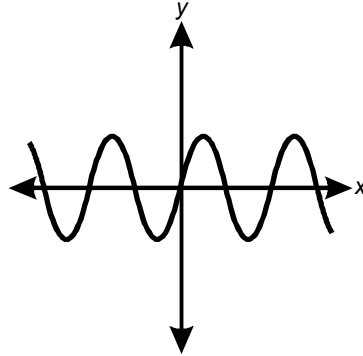
- A. no real roots.
- B. exactly one real root.
- C. exactly two real roots.
- D. exactly three real roots.

6. Which of the following is a polar coordinate graph of the form $r(\theta) = \cos n\theta$, where n is an integer?

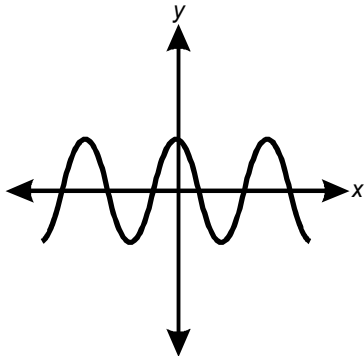
A.



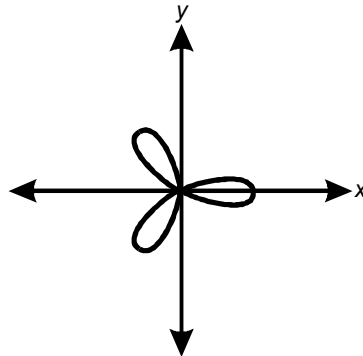
B.



C.

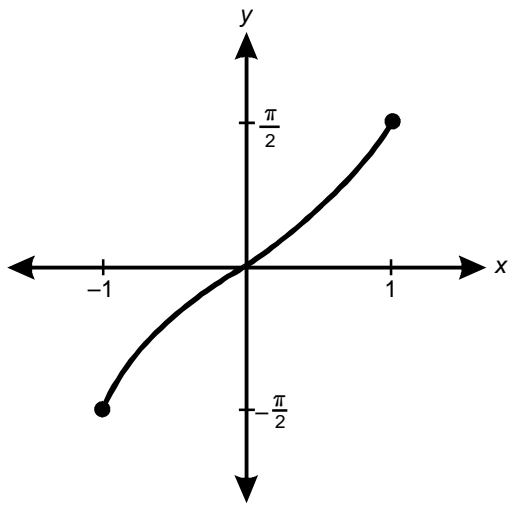


D.



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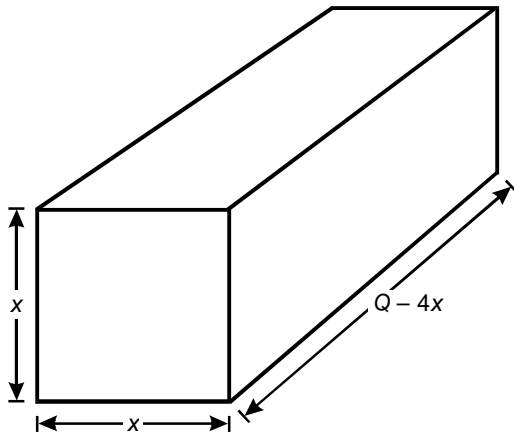
7. Use the graph below to answer the question that follows.



Given $f(x)$ above, which of the following functions is $f^{-1}(x)$, the inverse of $f(x)$, on the interval shown?

- A. $f^{-1}(x) = \sin x$
- B. $f^{-1}(x) = \cos x$
- C. $f^{-1}(x) = \sin^{-1} x$
- D. $f^{-1}(x) = \cos^{-1} x$

8. Use the diagram below to answer the question that follows.



The diagram above shows the measurements of a shipping carton. What value of x will maximize the volume of the carton?

- A. $x = \frac{Q}{6}$
- B. $x = \frac{Q}{4}$
- C. $x = \frac{\sqrt{Q}}{2}$
- D. $x = \sqrt{\frac{Q}{12}}$

ANNOTATED ANSWER KEY

For question	The correct response is	Reason	Test Objective
1	B	The factorial function $n!$ is defined as a product of consecutive positive integers. Since the set of positive integers is closed under multiplication, $r!$ and $p!$ must be integers. Since $0!$ is defined to be equal to 1, the expression $\frac{r!}{p!}$ is a quotient of two integers with division by zero excluded. This is the definition of a rational number.	0001
2	D	Raising a complex number in standard form to a power requires multiplying out several complex binomials. For example, $(a + bi)^4 = (a + bi)(a + bi)(a + bi)(a + bi) = a^4 + 4a^3(bi) + 6a^2(bi)^2 + 4a(bi)^3 + (bi)^4$. The right side of the equation then needs to be simplified back into standard form. Raising a complex number in trigonometric form to a power simply requires raising the magnitude to the power and multiplying the angle by the power. For example, $[r(\cos \theta + i \sin \theta)]^4 = r^4(\cos 4\theta + i \sin 4\theta)$. In this case, the right side of the equation is already in trigonometric form.	0002
3	B	If p is the period of the pendulum and ℓ is the length of the arm, then $p = k\sqrt{\ell}$ where k is the constant of proportionality. For $\ell = 4$ feet and $p = 2.2$ seconds, $2.2 = k\sqrt{4} = 2k$. Therefore, $k = 2.2 \div 2 = 1.1$.	0003
4	B	The driving time for each leg of the trip is the value in the row of the starting point and the column of the destination. Therefore, it takes 18 hours to drive from Atlanta to Boston, 16 hours from Boston to Chicago, 15 hours from Chicago to Denver, and 21 hours from Denver back to Atlanta. The total number of driving hours is $18 + 16 + 15 + 21 = 70$ hours.	0004
5	C	A function of the form $f(x) = ax^2 + bx + c$ is defined as a quadratic function if $a \neq 0$. The quantity $b^2 - 4ac$ is defined as the discriminant of the quadratic function. The discriminant determines the number of real roots as follows: if $b^2 - 4ac < 0$, then there are no real roots; if $b^2 - 4ac = 0$, then there is exactly one real root; and if $b^2 - 4ac > 0$, then there are exactly two real roots.	0005

For question	The correct response is	Reason	Test Objective
6	D	<p>On a polar graph, coordinates are in the form (r, θ), where angle θ is measured counterclockwise from the positive x-axis and distance r is measured from the origin. For $\theta = 0$, $r = \cos n(0) = \cos 0 = 1$.</p> <p>Hence the graph must pass through the point $(1, 0)$. As $n\theta$ increases to $\frac{\pi}{2}$ and θ increases to $\frac{\pi}{2n}$, r decreases to 0. As θ continues to increase, the value of r cycles between -1 and 1 forming a pattern of loops. The magnitude of r will never go beyond 1 and hence the graph will stay within the unit circle.</p>	0006
7	A	<p>Since $f(x)$ has a domain between -1 and 1 and a range between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, $f^{-1}(x)$ must have a domain between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ and a range between -1 and 1. Since $f(0) = 0$, $f^{-1}(0) = 0$. Of the choices given, the domain and range restrictions only fit the sine and cosine functions and the origin restriction only fits the sine function.</p>	0007
8	A	<p>The volume of a right rectangular prism is defined as the product of its length, width, and height. Therefore, the volume function $V(x) = (Q - 4x)(x)(x) = Qx^2 - 4x^3$. Setting the first derivative equal to zero to find the local extrema yields $V'(x) = 2Qx - 12x^2 = 2x(Q - 6x) = 0$. The solutions to this quadratic equation are $x = 0$ and $x = \frac{Q}{6}$. $V(0) = 0$. $V\left(\frac{Q}{6}\right) = \left(Q - \frac{4Q}{6}\right)\left(\frac{Q}{6}\right)^2 = \left(\frac{Q}{3}\right)\left(\frac{Q}{6}\right)^2 > 0$.</p> <p>Because the leading coefficient of the cubic volume function is negative, the volume decreases indefinitely for $x > \frac{Q}{6}$. The volume is undefined for $x < 0$. Therefore, the volume is maximized when $x = \frac{Q}{6}$.</p>	0008